Influence functional from a bath of coupled time-dependent harmonic oscillators

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(Received 31 July 1998)

The influence functional from a heat bath consisting of coupled harmonic oscillators of time-dependent frequency and coupling constants is derived. Like its time-independent counterpart, the present influence functional also involves the force-force autocorrelation function of the bath. [S1063-651X(99)12201-3]

PACS number(s): 05.90.+m

I. INTRODUCTION

Any system in real world is hardly separated from the environment. Due to the interaction with a large number of environmental degrees of freedom which act as a heat bath, the dynamics of the system becomes dissipative. In classical mechanics, the dynamics of the system is for practical purposes stochastic and is well described by the Langevin equation. By contrast, the quantum behavior of dissipative systems presents a more subtle problem and there exists no robust model that is easily quantized while capturing all known dynamic features in the classical limit [1,2]. At present, one of the most convenient approaches to quantum dissipative dynamics is the method of Feynman and Vernon [3,4], in which the effects of the heat bath enter into the reduced density matrix through an influence functional. These authors obtained a closed-form expression for the influence functional from an ensemble of independent harmonic oscillators interacting with the system of interest via a bilinear potential term. Their theory has been popularized by Caldeira and Leggett, who suggested that the harmonic oscillator bath provides a good approximation to real dissipative systems [5,6]. Many efforts have been made to establish a general quantum theory of dissipation; important approaches include the master equation developed by Zwanzig in the sixties [7], the Lindblad method [8], the quantum version of the Langevin equation [9], and a model-free analysis based on the fluctuation-dissipation theorem [10].

In all former studies the heat bath is assumed to be static. This is an exact picture in most cases where the environment is stable, as in the case of the phonons of a crystalline solid. It is also accurate in the linear response limit, that is, when the environment is adequately represented by a bath of fictitious harmonic oscillators characterized by an effective spectral density [11]. However, the static description may break down if the environment changes rapidly. A typical example is offered by processes in atomic or molecular liquids if the harmonic bath is defined in terms of the instantaneous normal modes of the fluid along a given classical trajectory [12].

Rather than attempting to tackle specific problems, the present paper is concerned with the influence functional from a dynamic heat bath. The composite Hamiltonian is assumed to be of the form

$$H(t)=H_0+H_b(t)+H_{\rm int}(t), \tag{1.1}$$

where H_0 is the Hamiltonian describing the system of interest, possibly including driving fields that are supposed not to act on the heat bath, $H_b(t)$ is the Hamiltonian of the dynamic heat bath, and $H_{\rm int}(t)$ is the system-bath interaction term. Note that the specific form of the system Hamiltonian is not relevant for the purpose of deriving the influence functional. In this article we focus on a dynamic bosonic bath consisting of harmonic oscillators whose frequencies change with time. This situation arises naturally in the instantaneous normal mode description of fluids [12]. Further, we choose a bilinear form for the interaction between the system of the bath, namely,

$$H_{\rm int}(t)=s\tilde{\mathbf{c}}(t)\cdot\mathbf{x},$$

where s is the coordinate of the system, $\mathbf{c}(t)$ is the time-dependent coupling vector, and $\mathbf{x}$ is the coordinate of the bath. In the above equation and throughout this paper the tilde denotes the transpose of a matrix.

Adopting the uncoupled initial condition of Caldeira and Leggett [6], we write the influence functional in the form

$$F[s^+,s^-]={\rm Tr}_b\{U_r[s^+]\rho_b(0)U_r^{-1}[s^-]\}. \tag{1.2}$$

Here $U_r[s]$ is the time evolution operator along a system path corresponding to the driven bath described by the Hamiltonian

$$H_r(t)\equiv H_b(t)+H_{\rm int}(s(t)) \tag{1.3}$$

and

$$\rho_b(0)=\frac{1}{Z}\,e^{-\beta H_b(0)} \tag{1.4a}$$

is the equilibrium density operator of the bath at the very instant that the system starts to interact with the bath and

$$Z={\rm Tr}_b\{e^{-\beta H_b(0)}\} \tag{1.4b}$$

is the quantum partition function.

In Sec. II we calculate the influence functional for a one-dimensional time-dependent harmonic bath. Section III deals with the multidimensional case where the bath is described by a force constant matrix which includes off-diagonal couplings. In Sec. IV we show that the kernel in the obtained expressions is given by the force autocorrelation function of the bath, ensemble averaged with respect to the initial density of the latter. A summary and brief concluding remarks are given in Sec. V.

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II. ONE-DIMENSIONAL TIME-DEPENDENT HARMONIC BATH

In mass weighted coordinates the Hamiltonian $H_r(t)$ reads

$$H_r(t)=\frac{p^2}{2}+\frac{1}{2}\,\omega^2(t)x^2-cs(t)x. \tag{2.1}$$

The propagator is

$$\begin{aligned}G(x,t;x',0)&\equiv\langle x|U_r[s]|x'\rangle\\&=N(t)\exp[-A(t)x^2-B(t)x'^2\\&\quad-C(t)xx'-D(t)x'-E(t)x],\end{aligned} \tag{2.2}$$

where

$$A(t)=-\frac{i}{2\hbar}\,\dot{R}_b(t)R_b^{-1}(t), \tag{2.3a}$$

$$B(t)=-\frac{i}{2\hbar}\,\dot{R}_a(t)R_b^{-1}(t), \tag{2.3b}$$

$$C(t)=\frac{i}{\hbar}\,R_b^{-1}(t), \tag{2.3c}$$

$$D(t)=\frac{i}{\hbar}\,u(t)R_b^{-1}(t), \tag{2.3d}$$

$$E(t)=-\frac{i}{\hbar}\int_0^t dt'f(t')R_b(t')R_b^{-1}(t), \tag{2.3e}$$

and the normalization constant is

$$\begin{aligned}N(t)=[2\pi i\hbar R_b(t)]^{-1/2}\exp\Bigg\{-\frac{i}{2}\Bigg[\dot{u}(t)u(t)\\-u^2(t)\dot{R}_b(t)R_b^{-1}(t)+\int_0^t dt'f(t')u(t')\Bigg]\Bigg\}.\end{aligned} \tag{2.3f}$$

In these formulas $f(t)=c(t)s(t)$ and $R_{a,b}(t)$ obey the homogeneous second-order differential equation

$$\ddot{R}_{a,b}(t)+\omega^2(t)R_{a,b}(t)=0 \tag{2.4}$$

with initial conditions

$$\begin{aligned}&R_a(0)=1,\quad \dot{R}_a(0)=0,\\&R_b(0)=0,\quad \dot{R}_b(0)=1.\end{aligned} \tag{2.5}$$

One sees that $R_{a,b}(t)$ are the trajectories of the classical harmonic oscillator with different initial settings. Similarly, the function $u(t)$ is determined by the inhomogeneous differential equation

$$\ddot{u}(t)+\omega^2(t)u(t)+f(t)=0 \tag{2.6}$$

with the initial condition $u(0)=0$ and $\dot{u}(0)=0$; that is, $u(t)$ is the trajectory of the driven harmonic oscillator described by $H_r(t)$, starting at rest.

To evaluate the influence functional, we exploit the invariance of the trace with respect to cyclic permutations and write Eq. (1.2) as

$$F[s^+,s^-]=\frac{1}{Z}\,\mathrm{Tr}_b\{U_r^{-1}[s^{-1}]U_r[s^+]e^{-\beta H_b(0)}\}. \tag{2.7}$$

In the one-dimensional case the partition function is

$$Z=\frac{1}{2\sinh[\hbar\beta\omega(0)/2]}.$$

It is convenient to calculate the trace over the heat bath in configuration space:

$$\begin{aligned}F[s^+,s^-]=\frac{1}{Z}\int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dx'\langle x|U_r^{-1}[s^-]U_r[s^+]|x'\rangle\\\times\langle x'|e^{-\beta H_b(0)}|x\rangle.\end{aligned} \tag{2.8}$$

The second factor in the integrand is the density matrix element for a static harmonic bath and is given by the expression

$$\begin{aligned}\langle x'|e^{-\beta H_b(0)}|x\rangle=\sqrt{\frac{\omega(0)}{2\pi\,\sinh[\hbar\beta\omega(0)]}}\\\times\exp\Bigg\{\frac{-\omega(0)}{2\sinh[\hbar\beta\omega(0)]}\\\times\{(x^2+x'^2)\cosh[\hbar\beta\omega(0)]-2xx'\}\Bigg\}.\end{aligned}$$

We evaluate the first factor using the known form of the propagator, namely,

$$\begin{aligned}&\langle x|U_r^{-1}[s^-]U_r[s^+]|x'\rangle\\&\quad=\int_{-\infty}^{\infty}dx_1G_-^*(x_1,t;x,0)G_+(x_1,t;x',0),\end{aligned}$$

where the subscripts $\pm$ refer to the harmonic oscillators driven by the external fields $s^{\pm}(t)$. Use of Eq. (2.2) leads to the result

$$\begin{aligned}&\langle x|U_r^{-1}[s^-]U_r[s^+]|x'\rangle\\&\quad=\frac{2\pi}{|iC(t)|}\,N_+(t)N_-(t)\exp[B(t)(x^2-x'^2)\\&\qquad+D_-(t)x-D_+(t)x']\,\delta(x-x'+y(t))\end{aligned} \tag{2.9}$$

where

$$y(t)=\int_0^t dt'[f_+(t')-f_-(t')]R_b(t')$$

and no indices are displayed if parameters in the propagator do not depend on external fields. By virtue of this result, one of the integrals in Eq. (2.8) is eliminated, while the second involves a Gaussian function and can be explicitly calculated. The resulting influence functional can be cast in the form of an exponential,

$$F[s^+,s^-]=\exp[\Phi(t)],$$

where the influence phase $\Phi(t)^3$ [3] is a complex function the real and imaginary parts of which read, respectively,

$$\mathrm{Re}\{\Phi(t)\}=-\frac{1}{4\hbar\,\omega(0)}[\omega^2(t)y^2(t)-z^2(t)]\times\coth[\hbar\beta\omega(0)/2] \tag{2.10}$$

and

$$\mathrm{Im}\ \Phi(t)=\frac{1}{2\hbar}\left\{\dot u_-(t)u_-(t)-\dot u_+(t)u_+(t)+\frac{\dot R_b(t)}{R_b(t)}[u_+^2(t)-u_-^2(t)]\right\}+\frac{1}{2\hbar}\int_0^t dt'[f_-(t')u_-(t')-f_+(t')u_+(t')]+\frac{i}{2}[D_+(t)+D_-(t)]y(t), \tag{2.11}$$

where

$$z(t)=\hbar(D_-(t)-D_+(t)-2y(t)B(t)). \tag{2.12}$$

To simplify this result we express the trajectories $u_\pm(t)$ of the driven system in terms of the solutions $R_{a,b}(t)$ of the homogeneous problem and the external fields $f_\pm(t)$. Note that in analogy with the equation of motion of the undriven harmonic oscillator

$$\ddot R(t)+\omega^2(t)R(t)=0$$

the Green's function is

$$G(t,t')=\frac{1}{\Delta}[\xi(t)\xi^*(t')-\xi^*(t)\xi(t')]\theta(t-t'), \tag{2.13}$$

where $\xi(t)=R_a(t)+iR_b(t)$, $\Delta=2i$ is the Wronskian, and $\theta(t-t')$ is the Heaviside step function. The trajectories of the driven harmonic oscillator are

$$u_\pm(t)=-\int_0^\infty dt' f_\pm(t')G(t,t')=\int_0^t dt' f_\pm(t')[R_a(t)R_b(t')-R_b(t)R_a(t')]. \tag{2.14}$$

Inserting the result into Eq. (2.13) and using also the formula of $y(t)$, we obtain

$$z(t)=i\int_0^t dt'[f_+(t')-f_-(t')]R_a(t'). \tag{2.15}$$

With these expressions, the real part of the influence phase Eq. (2.7) becomes

$$\mathrm{Re}\ \Phi(t)=-\frac{1}{2\hbar}\int_0^t dt'\int_0^t dt''[f_+(t')-f_-(t')]\times[f_+(t'')-f_-(t'')]M_1(t',t'',\beta), \tag{2.16}$$

where

$$M_1(t',t'',\beta)=\frac{1}{2\omega(0)}[\omega^2(0)R_b(t')R_b(t'')+R_a(t')R_a(t'')]\times\coth[\hbar\beta\omega(0)/2]. \tag{2.17}$$

Similarly, the imaginary part becomes

$$\mathrm{Im}\ \Phi(t)=-\frac{1}{\hbar}\int_0^t dt'\int_0^{t'} dt''[f_+(t')-f_-(t')]\times[f_+(t'')+f_-(t'')]M_2(t',t''), \tag{2.18}$$

where

$$M_2(t',t'')=\tfrac{1}{2}[R_a(t')R_b(t'')-R_b(t')R_a(t'')]. \tag{2.19}$$

Thus, the influence functional can be recast in the form similar to that of the conventional, static harmonic oscillator bath:

$$F[s^+,s^-]=\exp\left\{-\frac{1}{\hbar}\int_0^t dt'\int_0^{t'} dt''[s^+(t')-s^-(t')]\times[\alpha(t',t'')s^+(t'')-\alpha^*(t',t'')s^-(t'')]\right\}, \tag{2.20}$$

where the phase kernel is

$$\alpha(t',t'')=c(t')c(t'')[M_1(t',t'',\beta)+iM_2(t',t'')]. \tag{2.21}$$

Unlike the case of a static harmonic bath, the present kernel $\alpha(t',t'')$ is not a function of time displacement but depends on both time points. In Sec. IV we will prove that as in the time-independent case, $\alpha(t',t'')$ is nothing but the force autocorrelation function.

III. MULTIDIMENSIONAL BATH OF COUPLED TIME-DEPENDENT HARMONIC OSCILLATORS

In this section we extend the calculation of the influence functional to a multidimensional bath of coupled harmonic oscillators with time-dependent parameters. The Hamiltonian of the heat bath is

$$H_b(t)=\tfrac{1}{2}[\tilde{\mathbf{p}}\cdot\mathbf{p}+\tilde{\mathbf{x}}\cdot\omega^2(t)\cdot\mathbf{x}], \tag{3.1}$$

where $\mathbf{x}$ and $\mathbf{p}$ are coordinate and momentum vectors, and $\omega^2(t)$ is the force constant matrix, which is assumed to be symmetric without loss of generality.

In order to derive the influence functional in the multidimensional case we use the propagator recently derived by Macek in the presence of an external field [13]. The traditional method of deriving the propagator for a collection of uncoupled harmonic oscillators is not easily applicable to the coupled case. Macek postulated a Gaussian form and determined the coefficients of the various terms by comparing the resulting propagated wave function to the direct solution of the Schrödinger equation [13]. The result is

$$G(\mathbf{x},t;\mathbf{x}',0)\equiv\langle\mathbf{x}|U_r[s]|\mathbf{x}'\rangle = N(t)\exp(-\tilde{\mathbf{x}}\cdot\mathbf{A}(t)\cdot\mathbf{x}-\tilde{\mathbf{x}}\cdot\mathbf{B}(t)\cdot\mathbf{x}' -\tilde{\mathbf{x}}\cdot\mathbf{C}(t)\cdot\mathbf{x}'-\tilde{\mathbf{D}}(t)\cdot\mathbf{x}'-\tilde{\mathbf{E}}(t)\cdot\mathbf{x}), \quad (3.2)$$

where $N(t)$ is a (scalar) normalization constant, $\mathbf{A}(t)$, $\mathbf{B}(t)$, and $\mathbf{C}(t)$ are matrices, and $\mathbf{D}(t)$ and $\mathbf{E}(t)$ are vectors given by the equations

$$\mathbf{A}(t)=-\frac{i}{2\hbar}\dot{\mathbf{R}}_b(t)\cdot\mathbf{R}_b^{-1}(t), \quad (3.3a)$$

$$\mathbf{B}(t)=-\frac{i}{2\hbar}\dot{\mathbf{R}}_a(t)\cdot\mathbf{R}_b^{-1}(t), \quad (3.3b)$$

$$\mathbf{C}(t)=\frac{i}{\hbar}[\tilde{\mathbf{R}}_b(t)]^{-1}, \quad (3.3c)$$

$$\mathbf{D}(t)=\frac{i}{\hbar}\mathbf{R}_b^{-1}(t)\cdot\mathbf{u}(t), \quad (3.3d)$$

$$\mathbf{E}(t)=-\frac{i}{\hbar}[\tilde{\mathbf{R}}_b(t)]^{-1}\cdot\int_0^t dt'\tilde{\mathbf{R}}_b(t')\cdot\mathbf{f}(t') \quad (3.3e)$$

and

$$N(t)=\{\det[2\pi i\hbar\mathbf{R}_b(t)]\}^{-1/2} \times\exp\left\{-\frac{i}{2\hbar}\left(\dot{\tilde{\mathbf{u}}}(t)\cdot\mathbf{u}(t)-\tilde{\mathbf{u}}(t)\cdot\dot{\mathbf{R}}_b(t)\cdot\mathbf{R}_b^{-1}(t)\cdot\mathbf{u}(t) +\int_0^t dt'\tilde{\mathbf{f}}(t')\cdot\mathbf{u}(t')\right)\right\}, \quad (3.3f)$$

where the driving force is $\mathbf{f}(t)=s(t)\mathbf{c}(t)$, the square matrix functions $\mathbf{R}_{a,b}(t)$ are classical trajectories of the force-free harmonic bath that satisfy the equation

$$\ddot{\mathbf{R}}_{a,b}(t)+\omega^2(t)\cdot\mathbf{R}_{a,b}(t)=\mathbf{0} \quad (3.4)$$

with initial conditions

$$\mathbf{R}_a(0)=1,\quad \dot{\mathbf{R}}_a(0)=\mathbf{0},$$

$$\mathbf{R}_b(0)=\mathbf{0},\quad \dot{\mathbf{R}}_b(0)=1.$$

and the trajectory $\mathbf{u}(t)$ is determined by Newton's equation,

$$\ddot{\mathbf{u}}(t)+\omega^2(t)\cdot\mathbf{u}(t)+\mathbf{f}(t)=\mathbf{0}, \quad (3.5)$$

starting from a static position $\mathbf{u}(0)=\mathbf{0}$ and $\dot{\mathbf{u}}(0)=\mathbf{0}$. Note that both $\dot{\mathbf{R}}(t)\cdot\mathbf{R}^{-1}(t)$ and $\mathbf{B}(t)$ are symmetric matrices. Again, we employ the Green's function technique to obtain

$$\mathbf{u}(t)=\int_0^t dt'[\mathbf{R}_a(t)-\mathbf{R}_b(t)\cdot\mathbf{R}_b^{-1}(t')\cdot\mathbf{R}_a(t')]\cdot\tilde{\mathbf{R}}_b(t')\cdot\mathbf{f}(t'). \quad (3.6)$$

The derivation, though more tedious, is very similar to that performed with scalar functions in section II. To outline the main points, the forward-backward propagator matrix takes the form

$$\langle\mathbf{x}|U_r^{-1}[s^-]U_r[s^+]|\mathbf{x}'\rangle = \frac{2\pi N_+(t)N_-(t)}{|\det(i\tilde{\mathbf{C}}(t))|} \times\exp\{\tilde{\mathbf{x}}\cdot\mathbf{B}(t)\cdot\mathbf{x}-\tilde{\mathbf{x}}'\cdot\mathbf{B}(t)\cdot\mathbf{x}' +\tilde{\mathbf{D}}_-(t)\cdot\mathbf{x}-\tilde{\mathbf{D}}_+(t)\cdot\mathbf{x}'\}\delta(\mathbf{x}-\mathbf{x}'+\mathbf{y}(t)), \quad (3.7)$$

where

$$\mathbf{y}(t)=\int_0^t dt'\tilde{\mathbf{R}}_b(t')\cdot[\mathbf{f}_+(t')-\mathbf{f}_-(t')]. \quad (3.8)$$

When the Gaussian integration is completed, the influence functional is feasibly recast in exponential form $F[s^+,s^-]=\exp[\Phi(t)]$. The real and imaginary parts of the influence phase $\Phi(t)$ become, respectively,

$$\mathrm{Re}\,\Phi(t) =-\frac{1}{2\hbar}\int_0^t dt'\int_0^t dt''(\tilde{\mathbf{f}}_+(t')-\tilde{\mathbf{f}}_-(t')) \cdot\mathbf{M}_1(t',t'',\beta)\cdot(\mathbf{f}_+(t'')+\mathbf{f}_-(t'')), \quad (3.9)$$

and

$$\mathrm{Im}\,\Phi(t) =-\frac{1}{2\hbar}\int_0^t dt'\int_0^{t'} dt''(\tilde{\mathbf{f}}_+(t')-\tilde{\mathbf{f}}_-(t')) \cdot\mathbf{M}_2(t',t'',\beta)\cdot(\mathbf{f}_+(t'')+\mathbf{f}_-(t'')), \quad (3.10)$$

where

$$\mathbf{M}_1(t',t'',\beta)=\tfrac{1}{2}\mathbf{R}_b(t')\cdot\omega(0)\cdot\coth[\beta\hbar\omega(0)/2]\cdot\tilde{\mathbf{R}}_b(t'') +\tfrac{1}{2}\mathbf{R}_a(t')\cdot\omega^{-1}(0)\coth[\hbar\beta\omega(0)/2]\cdot\tilde{\mathbf{R}}_a(t'')$$

and

$$\mathbf{M}_2(t',t'')=\tfrac{1}{2}[\mathbf{R}_a(t')\cdot\tilde{\mathbf{R}}_b(t'')-\mathbf{R}_b(t')\cdot\tilde{\mathbf{R}}_a(t'')].$$

Since $\mathbf{M}_1$ is a symmetric matrix, the influence functional assumes the form

$$F[s^+,s^-]=\exp\left\{-\frac{1}{\hbar}\int_0^t dt'\int_0^{t'}dt''(s^+(t')-s^-(t'))\times[\alpha(t',t'')s^+(t'')-\alpha^*(t',t'')s^-(t'')]\right\}, \tag{3.11}$$

where the phase kernel is

$$\alpha(t',t'')=\tilde{\mathbf{c}}(t')\cdot(\mathbf{M}_1(t',t'',\beta)+i\mathbf{M}_2(t',t''))\cdot\mathbf{c}(t'')\equiv\tilde{\mathbf{c}}(t')\cdot\mathbf{M}(t',t'',\beta)\cdot\mathbf{c}(t''). \tag{3.12}$$

In the above equation the temperature dependence of the kernel has been suppressed. According to these results the influence functional for a multidimensional time-dependent harmonic bath takes the same form as that for a one-dimensional oscillator.

IV. PHASE KERNEL AND FORCE AUTOCORRELATION FUNCTION

Since the phase kernel $\alpha(t',t'')$ is seen to contain all information about the couplings to the bath, one expects this function to be related to the force the system experiences. In the case of a static bath, $\alpha(t',t'')$ is equal to the autocorrelation function of that force [3,4]. As shown below, this is also true of a time-dependent bath. From Eq. (3.11) one readily works out the second-order functional derivative, namely,

$$\left.\frac{\delta^2 F}{\delta s^+(t')\,\delta s^+(t'')}\right|_{s^+,s^-=0}=-\frac{1}{\hbar^2}\{\theta(t'-t'')\alpha(t',t'')+\theta(t''-t')\alpha(t'',t')\}. \tag{3.13}$$

With the definition of the influence functional Eq. (1.2), taking the second functional derivative gives

$$\left.\frac{\delta^2 F}{\delta s^+(t')\,\delta s^+(t'')}\right|_{s^+,s^-=0}=-\frac{1}{\hbar^2}\frac{1}{Z}\mathrm{Tr}_b\{\hat{T}(\tilde{\mathbf{c}}(t')\cdot\mathbf{x}(t')\tilde{\mathbf{c}}(t'')\cdot\mathbf{x}(t''))e^{-\beta H(0)}\}$$
$$=-\frac{1}{\hbar^2}\theta(t'-t'')\langle\tilde{\mathbf{c}}(t')\cdot\mathbf{x}(t')\tilde{\mathbf{c}}(t'')\cdot\mathbf{x}(t'')\rangle-\frac{1}{\hbar^2}\theta(t''-t')\langle\tilde{\mathbf{c}}(t')\cdot\mathbf{x}(t'')\tilde{\mathbf{c}}(t'')\cdot\mathbf{x}(t')\rangle, \tag{3.14}$$

where $\hat{T}$ is the chronological time-ordering operator, the brackets denote the ensemble average with respect to the initial density matrix, and the coordinate operators are in the interaction representation. Comparing the two formulas, one recognizes that

$$\alpha(t',t'')=\langle\tilde{\mathbf{c}}(t')\cdot\mathbf{x}(t')\tilde{\mathbf{c}}(t'')\cdot\mathbf{x}(t'')\rangle. \tag{3.15}$$

Recall that the system-bath coupling term is $H_{\mathrm{int}}(t)=s\tilde{\mathbf{c}}(t)\cdot\mathbf{x}$. Thus the $\alpha(t',t'')$ enjoys a nice interpretation: it is the autocorrelation function of the force operating on the system by the bath. Note that the derived result holds for the most general influence functional of Gaussian form.

The real and imaginary parts of the influence functional correspond to the classical and quantum contributions, respectively, and display different features in temporal locality. For a static harmonic oscillator bath, it has been shown that the temporal nonlocality in the classical factor can be removed by introducing auxiliary fields, but quantum nonlocality defies such localization [14]. One may draw the same conclusion upon recognizing that the real phase kernel can be written as the product of two functions of one time argument.

V. SUMMARY

Using the propagator derived by Macek [13], we have derived the influence functional from a bath of coupled, time-dependent harmonic oscillators. It turns out that the influence functional is determined by the classical trajectory of the undriven harmonic oscillator and that the kernel of the phase functional is the force-force autocorrelation function. These features are similar to the static harmonic oscillator bath. Note, however, that the classical trajectory of the time-dependent oscillator defies an analytic solution. To implement the influence functional in practical problems one should resort to numerical or approximate solutions [15,16].

ACKNOWLEDGMENT

This work has been supported by the David and Lucile Packard Foundation.

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